Measurement Error Subarea Model: An Application of Farm Labor Parameters

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Outline

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Motivation

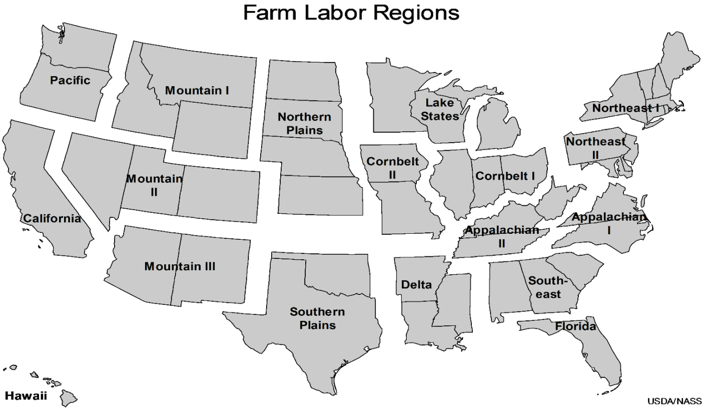
- Hierarchical Bayesian small area models are implemented in many NASS projects including Crops County Estimates, **Farm Labor**, and Cash Rents projects: NASEM (2018, 2023), Young and Chen (2022), Chen, et al. (2022a, 2022b, 2023).
- NASS contracted with NORC to conduct review and research improvements to NASS sampling methods, including for surveys resulting in small area estimation.
- One mid-term (2-4 years) recommendation of NORC's is to consider eliminating, reducing, or accounting for measurement error (ME) in the covariates in the current small area estimation modeling strategies.

Background

- Current models use the previous corresponding year's or quarter's official estimates.
- These covariates are subject to variability that would presumably differ among areas.
- Ignoring measurement error in small area models tends to be particularly problematic when the corresponding variances of the covariates measured with error differ among areas.
- The potential pitfalls include suboptimal prediction and incorrect estimation of uncertainty measures.
- Fuller (2009), Ybarra and Lohr (2007), Arima et al (2017), Bell et al. (2019).

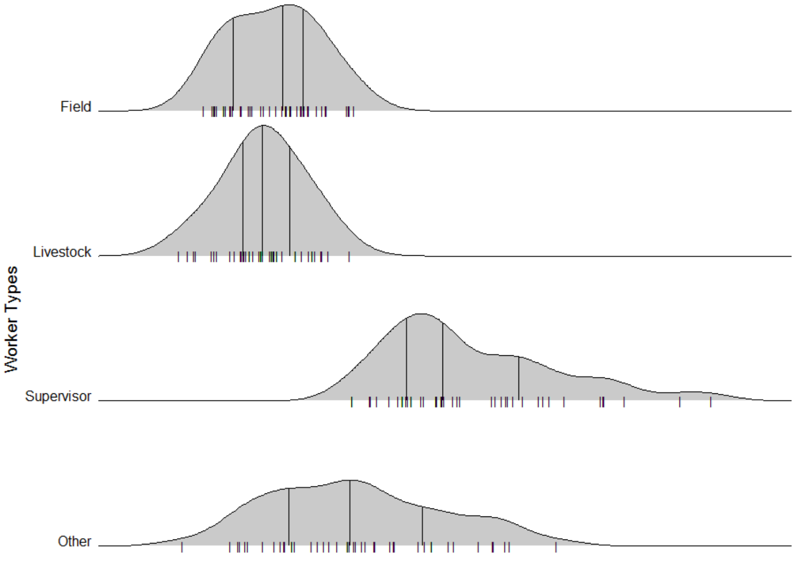
Data: Quantities of Interest

- Regional-level and US-level estimates:



- **NASS Worker Types**; the Standard Occupational Classification (SOC)

Direct Estimates — Wage Rates by Types



Notation

- $i = 1, \dots, m$ index for areas (i.e., regions)
- $j = 1, \dots, n_i$ index for subareas (i.e., states) within area i
- $\hat{\theta}_{ij},\ \hat{\sigma}^2_{ij}$ Farm Labor direct estimates by worker types
- x_{ij} known auxiliary information: the previous year, same quarter, official estimates; number of positive responses; and worker types

Subarea Model for Wage Rates (Original)

The subarea model for wage rates:

$$
\begin{gathered}
\hat{\theta}_{ij}|\theta_{ij} \overset{ind}{\sim} N(\theta_{ij},\ \hat{\sigma}^2_{ij}),\\
\theta_{ij}|\boldsymbol{\beta}, \nu_i, \sigma^2_\mu \overset{ind}{\sim} N(\mathrm{x}'_{ij}\boldsymbol{\beta} + \nu_i, \sigma^2_\mu), j = 1, \ldots, n_i,\\
\nu_i|\sigma^2_\nu \overset{iid}{\sim} N(0,\ \sigma^2_\nu),\ i = 1, \ldots, m,\\
\boldsymbol{\beta} \sim MN(\hat{\boldsymbol{\beta}},\ 1000 \times \hat{\Sigma}_{\hat{\beta}}),\\
\sigma^2_\mu{\sim}\text{Uniform}(R^+),\ \sigma^2_\nu{\sim}\text{Uniform}(R^+),
\end{gathered}
$$

- Goals:
  - State × type wage rate: $y^{wg}_{ijk} = \theta_{ijk}$
  - For publication: regional-level wage rates

$$
y^{wg,(h)}_k = \frac{\sum_{i=1}^m \sum_{j=1}^{n_i} y^{wk,(h)}_{ijk} y^{hr,(h)}_{ijk} y^{wg,(h)}_{ijk}}{\sum_{i=1}^m \sum_{j=1}^{n_i} y^{wk,(h)}_{ijk} y^{hr,(h)}_{ijk}},
$$

where $h = 1, ..., H$ are the draws and K are the worker types.

Conditional Structural Error Subarea Model

- One of the covariate $x_{ij1}(=\theta_{2ij})$ has measurement error, for example, previous estimates.
- Structural error model has non-identifiability issue for parameters.
- Proposed a two-part model and the two parts are connected via the multiplication rule of probability.
- Two subarea models connected by the non-identifiable parameter in the first part of the model:

  $$\pi(\theta_1, \theta_2 | D_1, D_2) = \pi(\theta_1 | \theta_2, D_1)\pi(\theta_2 | D_2),$$

  where D_1 and D_2 are the data from the two parts of the model.
- Given θ_2, all the parameters become identifiable in the first part of the model.

Conditional Structural Error Subarea Model

- The first part of the model $\pi(\theta_1|\theta_2, D_1)$:

$$\hat{\theta}_{1ij}|\theta_{1ij} \overset{ind}{\sim} N(\theta_{1ij},\ \hat{\sigma}^2_{1ij}),$$
$$\theta_{1ij}|\boldsymbol{\beta}_1, \theta_{2ij}, \nu_{1i}, \sigma^2_{\mu_1} \overset{ind}{\sim} N(\mathrm{x}'_{1ij}\boldsymbol{\beta}_1 + \gamma\theta_{2ij} + \nu_{1i}, \sigma^2_{1\mu}),$$

- The second part of the model $\pi(\theta_2|D_2)$:

$$\hat{\theta}_{2ij}|\theta_{2ij} \overset{ind}{\sim} N(\theta_{2ij},\ \hat{\sigma}^2_{2ij}),$$
$$\theta_{2ij}|\boldsymbol{\beta}_2, \nu_{2i}, \sigma^2_{\mu_2} \overset{ind}{\sim} N(\mathrm{x}'_{2ij}\boldsymbol{\beta}_2 + \nu_{2i}, \sigma^2_{2\mu}),$$

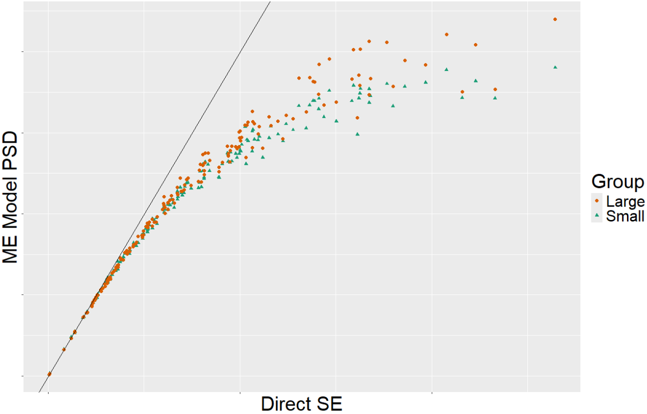
- The priors are similar to the original model.
- Note: $\hat{\theta}_{1ij}$ is the survey estimate and $\hat{\theta}_{2ij}$ is the covariate with measurement errors.

Case Study

- Example:
  - 44 states within 18 regions by worker types
  - Average wage rates
  - Two scenarios of measurement errors are checked:
    - Large variation: previous year's survey variances + noise related to sample sizes
    - Small variation: original model posterior variances based on the previous year survey

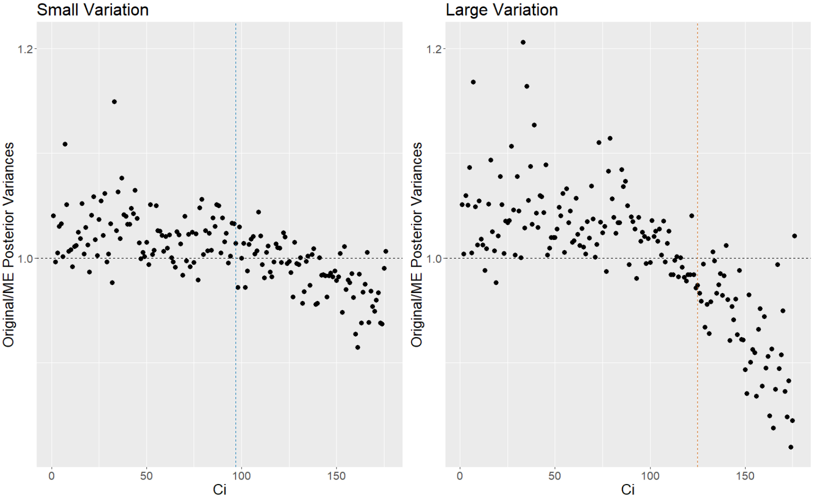
- Computation:
  - 15,000 samples and 5,000 burn-in, 3 chains, each thinned every 10 samples, resulting in a number of 3,000 samples for inference
  - Convergence diagnostics are conducted: Rhat ≤ 1.01 and effective sample sizes are around 3,000

Posterior Standard Deviation Comparisons

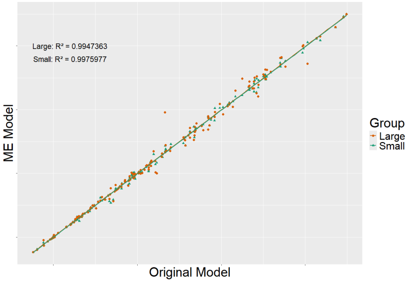


Posterior Variances Ratios v.s. Measurement Errors

Posterior Variances Ratios = Original / ME Posterior Variances



Posterior Mean Comparisons



$$\text{Absolute Relative Differences (\%)} = 100 \times \frac{|ME - Original|}{Original}$$

Cases	Min	25%	Median	Mean	75%	Max
Small	0.002	0.161	0.442	0.682	0.964	4.024
Large	0.005	0.185	0.477	0.962	1.090	14.500

Concluding Remarks

- Investigated the measurement error models from NORC's recommendation
- Proposed the conditional structural error model to avoid the non-identifiablity issue
- The current situation for the previous year's variations are with smaller variations
- However, with large variation, the precision differences are noticeable
- Both posterior means and posterior variances have large differences when the measurement errors are with large variations
- Further research and evaluation are needed

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Thank You!

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